

YOUR NAME: \_\_\_\_\_

George Voutsadakis

Read each problem **very carefully** before starting to solve it. Each problem is worth 10 points. It is necessary to show **all** your work. Correct answers without explanations are worth 0 points. GOOD LUCK!!

1. Compute the following derivatives:

(a)  $\left(\frac{x^3 - 1}{x^5 + x}\right)' =$

(b)  $[(x^3 + 9x)(5x + 3)^7]' =$

2. Find the tangent line to the graph of  $f(x) = \frac{1}{\sqrt[3]{(x^2 + x + 2)^2}}$  at  $x = 2$ .

3. Suppose that a particle moving on a straight line is situated at  $s(t) = (3t^2 + 2)^3$  meters from the origin at time  $t$  in seconds.

(a) Find an expression giving the velocity of the particle at time  $t$  seconds.

(b) Find an expression giving the acceleration of the particle at time  $t$  seconds.

4. A company produces personalized electronic gadgets. The cost per gadget is \$6.00 and the company has fixed costs of \$1,000. The owner has determined that, when  $x$  gadgets are produced, the price per gadget can be set at  $p(x) = 30 - 2x$  dollars.

(a) Find an equation for the cost, revenue and profit functions.

$$C(x) =$$

$$R(x) =$$

$$P(x) =$$

(b) Find an equation for the average profit function.

(c) Find an equation for the marginal average profit when 10 gadgets are produced and sold and interpret the answer.

5. Consider the function  $f(x) = 3x^4 - 8x^3 + 6x^2$ .

(a) Find its first derivative and its critical points(s).

(b) Construct the sign table for the first derivative and show clearly the intervals of monotonicity and the relative extrema of  $f$ .

(c) Use the information you gathered in Part (b) to sketch the graph of  $y = f(x)$ . (Please, be neat and label all your points!)