

YOUR NAME: _____

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Read each problem **very carefully** before starting to solve it. Each problem is worth 10 points. It is necessary to show **all** your work. Correct answers without explanations are worth 0 points. GOOD LUCK!!

1. An open-top box with a square base is to be constructed using materials that can afford surface area 192 square inches. Find the dimensions of the box that would maximize its volume.

2. A small brandy producer in Normandy, finds that it can sell 60 bottles of Calvados if it charges \$80 per bottle. It estimates that for each \$5 price increase, he will sell three fewer bottles. The cost per bottle is calculated to be \$30. Let x be the number of \$5 price increases that the producer will decide to make.

- (a) Find expressions for the price $p(x)$ and the quantity $q(x)$ as functions of x .

$$p(x) =$$

$$q(x) =$$

- (b) Find expression for the revenue $R(x)$, the cost $C(x)$ and the profit $P(x)$ in terms of x .

$$R(x) =$$

$$C(x) =$$

$$P(x) =$$

- (c) Find the price that should be charged per bottle in order to maximize the small outfit's profit.

3. Find an equation for the tangent line to the graph of $x^2 + y^2 = x^3y + 13$ at $(x, y) = (-1, 3)$.

4. A ice cube is melting, preserving its cubic shape. Suppose the volume of the cube is decreasing by 0.6 cubic inches per second. Find how fast the length of the cube is changing when the volume of the cube is 8 cubic inches.
5. (a) Solve the logarithmic equation $\log_3 x + \log_3 (x + 6) = 3$.
- (b) The population of coyotes in a restricted desert area is modeled by $P(t) = \frac{500}{1 + 1.5e^{-0.2t}}$, where t is in years since measurements begun.
- (i) Find how many coyotes where counted initially.
- (ii) Find after how many years there would be 400 coyotes inhabiting the area.