

YOUR NAME: _____

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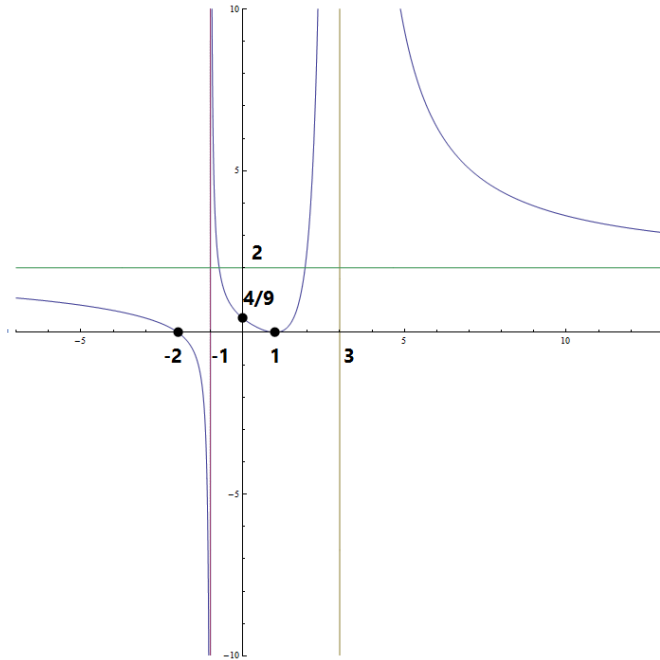
Read each problem **very carefully** before starting to solve it. Each problem is worth 10 points. It is necessary to show **all** your work. Correct answers without explanations are worth 0 points. GOOD LUCK!!

1. (a) Perform the long division $(x^4 + 3x^3) \div (x^2 + 2)$ and write your answer in the proper form.

- (b) The volume of a cylinder of radius r and height h is given by $V = \pi r^2 h$. If the volume is given by $V = \pi(3x^4 + 24x^3 + 46x^2 - 16x - 32)$ and the radius is $r = x + 4$, find the height h of the cylinder.

2. (a) Use the Remainder Theorem to find the remainder of the division $(3x^3 - 2x^2 + x - 4) \div (x + 3)$ without performing the division.
- (b) Use the Factor Theorem to find all zeros of the polynomial $f(x) = 2x^3 + 3x^2 + x + 6$, given that $x + 2$ is a factor.
- (c) A quartic (4th degree) polynomial with real coefficients has zeros $-3, 1$ and $-5i$. Find the polynomial if its leading coefficient is 1, and write it in the general form (sum of decreasing powers of x).

3. Consider the graph shown below.



(a) Find the domain $\text{Dom}(f)$.

(b) Find the vertical asymptotes.

(c) Find the horizontal asymptote.

(d) Find the x -intercepts.

(e) Find the y -intercept.

(f) Find a possible formula for the function $y = f(x)$ depicted in the graph. (Please, explain your thinking process and show all steps.)

4. (a) Consider the function $f(x) = (x + 1)^2 - 3$.

(i) Roughly sketch the graph of $y = f(x)$ (but show a few important points).

(ii) Restrict the domain so that f be one-to-one.

(iii) Find an inverse function for the one-to-one function you have created in Part (ii) and state its domain clearly.

(b) Find the inverse $f^{-1}(x)$ if $f(x) = \frac{2x + 5}{3 - x}$.

5. (a) The quantity z varies directly with both the square of x and the square root of y . If $z = 32$ when $x = 2$ and $y = 4$, find x when $y = 25$ and $z = 40$.

- (b) The quantity z varies directly with the square of x and the cube of y and inversely with the square root of w . Suppose that $z = 12$ when $x = 2$, $y = 2$ and $w = 64$. Find z if $x = 1$, $y = 3$ and $w = 81$.