EXAM 3 - MATH 112 Friday, November 5

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Read each problem **very carefully** before starting to solve it. Each problem is worth 10 points. It is necessary to show **all** your work. Correct answers without explanations are worth 0 points. GOOD LUCK!!

1. An open-top box with a square base is to have volume exactly 500 in^3. Our goal is to find the dimensions of the box that can be made with the smallest amount of materials.

   (a) Read the statement carefully, create a small drawing and set your variables.

   (b) Write an objective function reflecting the quantity we want to minimize.

   (c) If your objective function contains more than one variable, write an auxiliary function based on additional data provided in the statement. Solve the auxiliary for one of the variables.

   (d) Use the auxiliary to rewrite the objective function with a single variable and optimize to obtain the answer to the question posed in the statement.

2. A computer dealer can sell 12 personal computers per week at a price of \$2,000 each. He estimates that each \$400 decrease in price will result in three more sales per week. Each computer costs \$1,200. Let d be the number of \$400 decreases that he decides to make.

   (a) Write an equation for the price $p(d)$ as a function of d.

   (b) Write an equation for the number $q(d)$ of computers sold per week as a function of d.

   (c) Write equations for the revenue, cost and profit functions.

   $$R(d) =$$

   $$C(d) =$$

   $$P(d) =$$

   (d) Find the price that should be charged to maximize the profit.

3. Find an equation for the tangent line to the graph of $x^2 + y^2 = xy^2 + 1$ at the point $(x, y) = (3, 2)$.

4. A cube (all sides equal) of ice is melting so that its volume is decreasing at the rate of $\frac{1}{8}$ in^3 per hour. How fast is each side decreasing, when each side is 2 inches long?

5. (a) Compute the equation for the tangent line to $f(x) = x^3 e^{-x}$ at $x = -1$.

(b) Consider the function $f(x) = \ln\left(\frac{x^3(x-7)^2}{x^2+5}\right)$.

(i) Use the properties of logarithms to break $f(x)$ as a sum/difference of multiples of simpler logarithms.

(ii) Use the expression you obtained in Part (i) to compute $f'(x)$.