Read each problem **very carefully** before starting to solve it. Each problem is worth around 5 points. It is necessary to show **all** your work. Correct answers without explanations are worth 0 points. GOOD LUCK!!

1. [4 points] A bike company finds that it costs $200 to produce a bike and that fixed costs amount to $1500 per day. A sales analysis shows that the price function is $p(x) = 600 - 5x$, where p is the price in dollars per bike at which exactly x bikes will be sold.

   (a) Find equations for the cost, revenue and profit functions.

$$C(x) \;=\;$$

$$R(x) \;=\;$$

$$P(x) \;=\;$$

   (b) Find the number of bikes that should be sold and the price that should be charged per bike to maximize the company's profit.

2. [5 points] A farmer wants to make three identical enclosures along a straight river as shown below. If he has 1200 yards of fencing available and the sides along the river need no fencing, what should the dimensions of each enclosure be so that the total area is maximized?
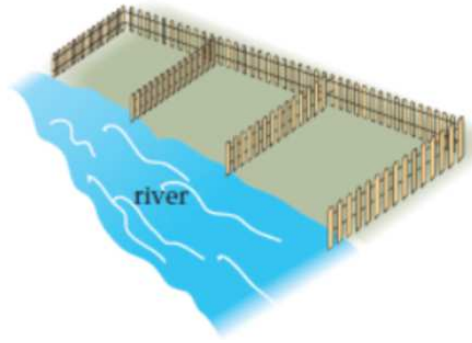
(a) Find an objective function.

(b) Write an auxiliary equation if you need to eliminate a variable from the objective function.

(c) Perform the optimization step to answer the question.

3. [6 points] A dealership can sell eight cars per day at $24,000 each. For each $800 reduction in price, it can sell four more cars per day. Each car costs $20,000 and the fixed costs per day are $2,000. Let d be the number of $800 reductions the dealer decides to make.

(a) Find an equation $p(d)$ of the price as a function of d.

(b) Find an equation for the number $q(d)$ of cars sold as a function of d.

(c) Find an equation for the cost $C(d)$.

(d) Find an equation for the revenue $R(d)$.

(e) Find an equation for the profit $P(d)$.

(f) Which price should be charged per car to maximize the dealership's profit?