

YOUR NAME: \_\_\_\_\_

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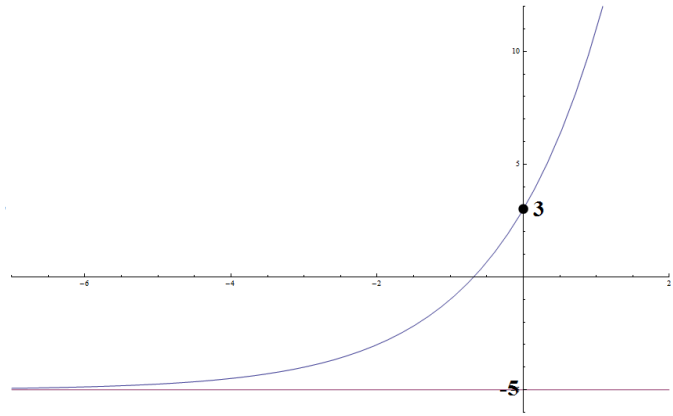
Read each problem **very carefully** before starting to solve it. Each problem is worth 10 points. It is necessary to show **all** your work. Correct answers without explanations are worth 0 points. GOOD LUCK!!

1. The population of a certain species in 2019 was 4,000. The population rose to 5,234 by 2022.

(a) Assuming an exponential growth, find a model for the population as a function of time.

(b) Assuming that the same trend will continue, estimate (showing all steps) when the population will reach 20,000 individuals.

2. (a) Sketch below on the left the graph of  $y = 2^x$  by using a small table of values. Label all points.



- (b) Find a formula for a function  $y = f(x)$  whose graph is shown on the right above, assuming that its parent function is  $y = 2^x$ . Justify all choices. (Hint: Do not use both a vertical stretch and a horizontal shift; one of the two suffices!)
3. Solve the following equations by hand and give the exact value of the solutions (that is, do not use your calculators to compute any decimals).

(a)  $3 \cdot 7^{2x+1} - 10 = 50$ .

(b)  $\log_3(5 - 2x) = 4$ .

4. (a) Sketch the graph of  $f(x) = \log_{1/3}(x)$  using a small table of values. Label all points.

(b) Detail all transformations that lead from  $f(x) = \log_{1/3}(x)$  to  $g(x) = -2\log_{1/3}(x + 5) - 3$ .

$$f(x) = \log_{1/3}(x) \longrightarrow ( \quad \quad )$$

$$\longrightarrow ( \quad \quad )$$

$$\longrightarrow ( \quad \quad )$$

$$\longrightarrow g(x) = -2\log_{1/3}(x + 5) - 3 \quad ( \quad \quad )$$

5. (a) Solve the exponential equation

$$8^{3x-47} = 16^{3-2x}.$$

(b) Solve the logarithmic equation

$$\log_7(x) + \log_7(x - 6) = 1.$$