

YOUR NAME: _____

George Voutsadakis

Read each problem **very carefully** before starting to solve it. Each problem is worth 10 points. It is necessary to show **all** your work. Correct answers without explanations are worth 0 points. GOOD LUCK!!

1. (a) Express the point $(10, -5)$ in polar coordinates.

- (b) Express the equation $4x + 5y = 100$ in polar coordinates and write it in the form $r = r(\theta)$.

- (c) Express the equation $r = 3 \cos \theta$ in cartesian coordinates, without any trig or inverse trig functions.

2. (a) Write the complex number $z = 5\sqrt{3} + 5i$ in polar form.

(b) Compute the complex cube roots of $8i$ and leave your answers in standard form.

3. (a) Eliminate the parameter from the system of parametric equations $\begin{cases} x = 5 \sin t \\ y = 7 \cos t \end{cases}$ to obtain an equation in rectangular coordinates containing no trig or inverse trig functions.

- (b) Write $x + y^2 = 3$ as a system of parametric equations, showing all steps.

4. Consider the vectors \mathbf{v} with initial point $(1, 2)$ and terminal point $(-2, 4)$ and \mathbf{u} with initial point $(-3, 5)$ and terminal point $(4, 6)$.

(a) Write the vectors \mathbf{v} and \mathbf{u} as position vectors.

(b) Compute the vector $5\mathbf{u} - 3\mathbf{v}$, showing all steps.

(c) Find a unit vector in the direction of \mathbf{u} .

(d) Calculate the dot product $\mathbf{v} \cdot \mathbf{u}$.

(e) Find the angle θ between the two vectors in degrees approximated to 2 decimal digits.

5. For those that have not taken any physics, the total force applied on an object is the vector sum of the individual forces applied on it.

Suppose that an object is located at the origin and that the following two forces are applied on it.

- A force of size 100 N (Newtons) that forms an angle of 45° with the positive x -axis;
- A force of size 150 N that forms an angle of -30° with the positive x -axis.

Find the magnitude and the direction of the total force that is applied on the given object. (Try to give exact values whenever possible.)