

YOUR NAME: _____

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Read each problem **very carefully** before starting to solve it and do only what is asked. Each problem is worth around 5 points. It is necessary to show **all** your work. Correct answers without explanations are worth 0 points. GOOD LUCK!!

1. [4 points] Starting from the formula for $\cos(\alpha + \beta)$, give the following, showing all steps:

(a) A formula expressing $\cos(2\theta)$ exclusively in terms of $\cos \theta$.

(b) A formula expressing $\cos\left(\frac{\theta}{2}\right)$ exclusively in terms of $\cos \theta$.

(c) Use the formula obtained in Part (b) to calculate the exact value of $\cos\left(\frac{7\pi}{8}\right)$.

2. [4 points]

(a) If $\sin \theta = -\frac{3}{7}$ and θ lies in Quadrant III, compute the exact values of $\sin(\frac{\theta}{2})$ and $\cos(\frac{\theta}{2})$.

(b) Verify the following identity

$$\cos(3x) = \cos^3 x - 3 \sin^2 x \cos x.$$

3. [4 points]

(a) Convert a product to a sum or vice-versa, as appropriate, but do not evaluate:

(i) $\cos(23^\circ)\sin(17^\circ) =$

(ii) $\sin(76^\circ) + \sin(14^\circ) =$

(b) Prove that $\sin(x) + \sin(3x) = 4\sin(x)\cos^2(x)$.

4. [4 points] Solve the following equations, where $0 \leq \theta < 2\pi$:

(a) $2 \sin(2\theta) = \sqrt{3}$.

(b) $2 \sin \theta \cos \theta - \sin \theta + 2 \cos \theta - 1 = 0$ (Hint: Find a way to factor.)

(c) $8 \sin^4 \theta - 6 \sin^2 \theta + 1 = 0$ (Hint: Use substitution.)