EXAM 3 - MATH 310 Friday, November 4

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Read each problem **very carefully** before starting to solve it. Each problem is worth 10 points. It is necessary to show **all** your work. Correct answers without explanations are worth 0 points. GOOD LUCK!!

1. Find the general solution of the differential equation

$$y^{(4)} + 7y''' + 12y'' + 14y' + 20y = 0.$$

Assume that we are given that $r = -5$ is one of the roots of its characteristic equation.

2. Find the general solution of

$$y''' + 4y' = 16t + 4.$$

3. (a) Suppose we know that $\mathcal{L}(y') = sF(s) - y(0)$, where $F(s) = \mathcal{L}\{y\}$. Show, based on this, that

$$\mathcal{L}\{y''\} = s^2F(s) - sy(0) - y'(0).$$

(b) Compute from scratch the Laplace transform of $f(t) = u_3(t) - u_7(t)$.

4. Compute the Laplace transform $F(s)$ of the solution $f(t)$ of the initial value problem

$$y'' - y' = u_3(t) - u_1(t), \quad y(0) = 2,\ y'(0) = 0.$$

5. Compute the inverse Laplace transform of $F(s) = \dfrac{e^{-5s}(s+11)}{s^2 - 6s + 58}$.