

YOUR NAME: _____

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Read each problem **very carefully** before starting to solve it. Each problem is worth 10 points. It is necessary to show **all** your work. Correct answers without explanations are worth 0 points. GOOD LUCK!!

1. (a) Sketch the graph of the function $d_{1/4}(t - \frac{1}{2})$.

- (b) Compute from scratch the Laplace transform $\mathcal{L}\{d_{1/4}(t - \frac{1}{2})\}$.

2. Find the particular solution of

$$y'' - 2y' - 3y = \delta\left(t - \frac{1}{2}\right), \quad y(0) = 0, \quad y'(0) = 0.$$

3. Compute from scratch $(f * g)(t)$ if $f(t) = t$ and $g(t) = e^{3t}$.

4. Find the general solution of

$$y'' + 3y = g(t), \quad y(0) = 5, y'(0) = -1.$$

5. Find the general solution of the homogeneous system of linear differential equations

$$\mathbf{y}'(t) = \begin{pmatrix} 1 & 1 \\ 6 & 2 \end{pmatrix} \mathbf{y}(t), \text{ where } \mathbf{y}(t) = \begin{pmatrix} y_1(t) \\ y_2(t) \end{pmatrix}.$$