

EXAM 3 - MATH 152

Friday, November 10

YOUR NAME: \_\_\_\_\_

George Voutsadakis

Read each problem **very carefully** before starting to solve it. Each problem is worth 10 points. It is necessary to show **all** your work. Correct answers without explanations are worth 0 points. GOOD LUCK!!

1. Find an exact expression (no decimals) for the Taylor polynomial  $T_3(x)$  of

$$f(x) = x^2 e^{-x}$$

centered at  $a = 1$ .

2. Solve the initial value problem

$$yy' = (t + 1)(y^2 + 1), \quad y(0) = 4.$$

3. Decide whether the following sequences converge or diverge. You should state clearly which criterion you are using and how you are applying it.

(a)  $a_n = \frac{\sqrt{n}}{\sqrt{n} + 7}$

(b)  $b_n = \frac{2^n}{n!}$ .

4. (a) Find the sum of the series  $\sum_{n=0}^{\infty} \frac{3(-2)^n - 5^{n+1}}{8^n}$

- (b) Tell whether the following series is absolutely convergent, conditionally convergent or divergent, showing all steps and justifications.  $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{\sqrt[5]{n^4}}$ .

5. Use whichever of the ratio or root test is appropriate to investigate convergence or divergence of the following series. Show all calculations and justifications.

(a)  $\sum_{n=1}^{\infty} \frac{e^n}{n^n}$

(b)  $\sum_{n=1}^{\infty} \frac{n!}{e^n}$