

EXAM 4 - MATH 152

Friday, December 1

YOUR NAME: _____

George Voutsadakis

Read each problem **very carefully** before starting to solve it. Each problem is worth 10 points. It is necessary to show **all** your work. Correct answers without explanations are worth 0 points. GOOD LUCK!!

1. Find the center, the radius of convergence and the interval of convergence of the power series

$$\sum_{n=1}^{\infty} \frac{(-1)^n n}{4^n} (x+3)^n.$$

2. Consider the function $f(x) = \frac{1}{1-x^3}$.

(a) Find a power series for $f(x)$ (centered at $a = 0$).

(b) Compute the derivative $f'(x)$.

(c) Find a power series for the function $\frac{3x^2}{(1-x^3)^2}$.

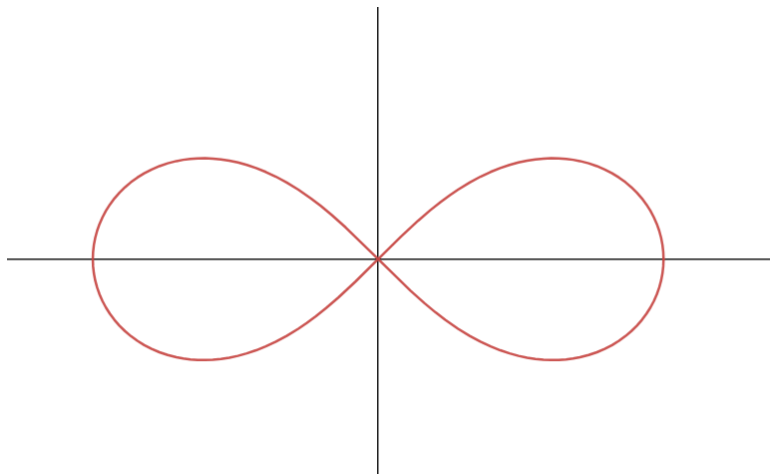
3. Consider the system of parametric equations

$$\begin{cases} x = \sin^2 t \\ y = \cos^2 t \end{cases}, \quad 0 \leq t \leq \frac{\pi}{2}.$$

(a) Find an equation for the tangent line to its graph at $t = \frac{\pi}{6}$.

(b) Compute the surface area of the solid resulting from revolving the graph around the x -axis.

4. Find the points on the graph of the polar curve $r^2 = \cos 2\theta$, where the tangent line is horizontal. (**Hint:** You may find the formula $\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$ useful.)



5. Find the area of the shaded region depicted below. Show all steps!

