EXAM 1 - MATH 310 Friday, September 22

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Read each problem **very carefully** before starting to solve it. Each problem is worth 10 points. It is necessary to show **all** your work. Correct answers without explanations are worth 0 points. GOOD LUCK!!

1. Solve the initial value problem

$$y' + y^2 \sin x = 0, \quad y\left(\frac{3\pi}{2}\right) = \frac{1}{8}.$$

2. (a) Use the by-parts method to compute the integral

$$\int x \sin x dx.$$

(b) Find the general solution of the differential equation

$$ty' + 2y = \sin t, \quad t > 0.$$

3. A 100-gallon tank initially contains 1 gallon of pure water. Starting at $t_0 = 0$, water that contains $\frac{1}{t+1}$ lbs of salt per gallon is poured into the tank at the rate of 2 gallons/min and the mixture is drained from the tank at the rate of 1 gallon/min. Find the amount of salt $Q(t)$ in the tank at time t, where $1 \leq t \leq 99$ minutes.

4. Check whether the given equation is exact and, if yes, solve it.

$$(9x^2 + y - 1) - (4y - x)y' = 0.$$

5. Solve the initial value problem

$$2y'' + 9y' - 5y = 0, \quad y(0) = 8, \quad y'(0) = \frac{25}{2}.$$