EXAM 4 - MATH 310 Friday, December 1

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Read each problem **very carefully** before starting to solve it. Each problem is worth 10 points. It is necessary to show **all** your work. Correct answers without explanations are worth 0 points. GOOD LUCK!!

1. Find the particular solution of

$$y'' + y = g(t), \quad y(0) = 0, \quad y'(0) = 0,$$

where $g(t) = \begin{cases} \sin t, & \text{if } 0 \leq t < \pi \\ 0, & \text{if } t \geq \pi \end{cases}$.

2. Find the particular solution of

$$y'' + 4y' = \delta(t - 2\pi)\cos t, \quad y(0) = 1, \quad y'(0) = 0.$$

3. Express the solution of the initial value problem in terms of a convolution integral.

$$y'' + 3y' + 2y = g(t), \quad y(0) = 1, \quad y'(0) = 2.$$

4. Consider the following system of linear equations:

$$\left\{ \begin{array}{rcrcr} -7x_1 & - & 3x_2 & = & 6 \\ 5x_1 & + & 2x_2 & = & -5 \end{array} \right\}.$$

(a) Write the system as a matrix equation $\mathbf{A}\mathbf{x} = \mathbf{b}$.

(b) Find the inverse matrix $\mathbf{A}^{-1}$ by hand.

(c) Multiply on the left by $\mathbf{A}^{-1}$ to solve the system for $\mathbf{x}$.

5. Find the particular solution of the homogeneous system of linear first-order differential equations with constant coefficients.

$$\mathbf{x}' = \begin{pmatrix} 4 & -3 \\ 2 & -3 \end{pmatrix} \mathbf{x}, \quad \mathbf{x}(0) = \begin{pmatrix} -1 \\ 3 \end{pmatrix}.$$