Friday, September 15

Read each problem very carefully before starting to solve it and do only what is asked. Each problem is worth around 5 points. It is necessary to show all your work. Correct answers without explanations are worth 0 points. GOOD LUCK!!

1. [6 points] At time $t=0$ a tank contains 40 lb of salt dissolved in 100 gallons of water. Water containing $\frac{1}{10} \mathrm{lb}$ of salt/gallon is entering the tank at a rate of 1 gallon/minute and the mixture is draining from the tank at the rate of 2 gallons/minute.
(a) Set up the initial value problem that describes this flow process.
(b) Solve the equation to find the amount of salt $Q(t)$ in the tank at time $t$. (Note that your equation will only be valid for $0 \leq t \leq 100$ minutes.)
(c) Find how much salt will be in the tank when the tank is half-full (or half-empty).
2. [6 points] Consider a population $p(t)$ of certain species in a certain area counted in number of individuals at time $t$ in years after measurements began. The rate of increase is proportional to the current population with constant of proportionality $\frac{1}{4}$, while, because of a certain disease, the rate of increase is tempered by a loss rate of $e^{-\frac{1}{10} t}$ (depending on time, but not on the population).
(a) Write an initial value problem modeling the rate of change of the population, given that, initially, the population consisted of 10 individuals.
(b) Solve the initial value problem to find the population $t$ years after measurements began.
