Friday, November 8 George Voutsadakis

Read each problem **very carefully** before starting to solve it. Each problem is worth 10 points. It is necessary to show **all** your work. Correct answers without explanations are worth 0 points. GOOD LUCK!!

1. Solve the initial value problem

$$\frac{dy}{dx} = 2x - xy, \quad y > 2, \quad y(0) = 5.$$

Please, give the solution in explicit form.

2. Use the comparison or limit comparison tests to decide convergence or divergence of the following series (show all details):

(a) 
$$\sum_{n=1}^{\infty} \frac{\sqrt{n}}{4n+9}.$$

(b) 
$$\sum_{n=1}^{\infty} \frac{4^n}{5^n - 2n}$$
.

3. Decide whether the following series converges absolutely, conditionally or diverges. (Show all details.)

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n^2+1}}$$

4. Use the ratio or the root test to decide convergence or divergence of the following series (show all details):

(a) 
$$\sum_{n=0}^{\infty} \left(\frac{n}{3n+5}\right)^n$$
.

(b) 
$$\sum_{n=1}^{\infty} \frac{(-1)^{n-1}n}{3^n}.$$

5. Find the center, the radius of convergence and the interval of convergence of the power series

$$\sum_{n=1}^{\infty} \frac{3^n}{4n} (x-2)^n.$$

(Show all details.)