QUIZ 7 - MATH 310 Thursday, October 31

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Read each problem **very carefully** before starting to solve it and do only what is asked. Each problem is worth around 5 points. It is necessary to show **all** your work. Correct answers without explanations are worth 0 points. GOOD LUCK!!

1. [4 points] Consider the function

$$f(t) = \begin{cases} 2t, & \text{if } 0 \leq t < 1. \\ 1, & \text{if } t \geq 1. \end{cases}$$

   (a) Sketch the graph $y = f(t)$.

   (b) Compute from scratch the Laplace transform $\mathcal{L}\{f\}$.

2. [4 points] Find the Laplace (in Part (a)) or inverse Laplace (in Part (b)) transforms of the following:

(a) $f(t) = 3e^{4t} - 7e^{2t}\cos 3t + 5e^{-t}\sin\frac{1}{2}t.$

(b) $F(s) = \dfrac{2s+1}{s^2+4} - \dfrac{7}{s+3}.$

2. [6 points] Use Laplace transforms to solve the initial value problem

$$y'' - 4y' - 5y = 0, \quad y(0) = 1, \quad y'(0) = 0.$$

(a) Phase 1: Find $F(s) = \mathcal{L}\{y\}$.

(b) Phase 2: Decompose $F(s)$ into partial fractions.

(c) Phase 3: Apply the inverse Laplace transform to find $y = f(t) = \mathcal{L}^{-1}\{F(s)\}$.