EXAM 3 - MATH 112 Friday, March 26

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Read each problem **very carefully** before starting to solve it. Each problem is worth 10 points. It is necessary to show **all** your work. Correct answers without explanations are worth 0 points. GOOD LUCK!!

1. Consider the function $f(x) = x^4 - 32x$.

   (a) Compute $f'(x)$ and find its critical point(s).

   (b) Compute $f''(x)$ and find its critical point(s).

   (c) Create the combined sign table for f' and f'' and draw conclusions about f regarding monotonicity, concavity, relative extrema and inflection points. (Please, no separate tables for f' and f''; I would like to see the combined sign table as shown in class, with curved arrows in the last line.)

2. In this problem I will guide you in creating an open-top box with a square base, having volume 108 cubic feet, using the least amount of materials.

   (a) Assume that the box has base side x feet and height y feet. Write an equation for the volume of the box in terms of x and y. Then solve it for y.

   (b) Write an equation for the surface area of the box in terms of both x and y and use the equation of Part (a) to eliminate y.

   (c) Use the equation of Part (b) to find the size of x that minimizes the surface area of the box.

3. Find an equation for the tangent line to

$$x^2y - y + 1 = 2x^3$$

at $(x, y) = (2, 5)$.

4. A spherical bubble is inflated at the rate of 9π cubic inches per second. Find how fast its radius is changing, when the radius is $\frac{3}{4}$ inches.

(The volume of a sphere as a function of its radius is given by $V = \frac{4}{3}\pi r^3$.)

5. (a) Compute the following derivatives:

$(x^2 e^x)' =$

$\left[\ln\left(x^5 - 3x^2 + 1\right)\right]' =$

(b) Find an equation for the tangent line to $f(x) = e^{x^2}$ at $x = 1$.