Read each problem very carefully before starting to solve it. Each problem is worth around 5 points. It is necessary to show all your work. Correct answers without explanations are worth 0 points. GOOD LUCK!!

1. Consider the function $f(x)=2 x^{4}-8 x^{3}+30$.
(a) Compute $f^{\prime}(x)$ and find critical points.
(b) Compute $f^{\prime \prime}(x)$ and find critical points.
(c) Create the combined sign table for $f^{\prime}$ and $f^{\prime \prime}$ and draw conclusions about monotonicity, relative extrema, concavity and inflection points.
2. Consider $f(x)=x+\frac{9}{x}$. Use the second derivative test to find the relative extrema of $f$. (Hint: To find the critical points of $f^{\prime}$ write it as a single fraction.)
3. A certain company produces a gadget. The cost per gadget produced in $\$ 3,000$ and the fixed costs are $\$ 10,000$. Moreover, the price the company charges per gadget is $p(x)=7000-10 x$, where $x$ is the number of gadgets produced and sold.
(a) Write an expression for the company's cost function $C(x)$.
(b) Write an expression for the company's revenue function $R(x)$.
(c) Write an expression for the company's profit function $P(x)$ and simplify it.
(d) Find the number of gadgets that need to be sold and the price at which they should be sold to maximize the company's profit.
