Read each problem **very carefully** before starting to solve it. Each problem is worth 10 points. It is necessary to show **all** your work. Correct answers without explanations are worth 0 points. GOOD LUCK!!

1. (a) Write in terms of a single trigonometric function

$$\cos\frac{3x}{4}\cos\frac{x}{4} + \sin\frac{3x}{4}\sin\frac{x}{4} =$$

(b) Suppose $\sin\alpha = \dfrac{3}{5}$, α in Quadrant I, and $\tan\beta = \dfrac{5}{12}$, β in Quadrant III. Compute $\cos(\alpha - \beta)$.

2. (a) Suppose $\sin \alpha = -\dfrac{3}{7}$ and $270° < \alpha < 360°$. Calculate $\cos \frac{\alpha}{2}$.

(b) Verify the identity $\sin 2x - \cot x = -\cot x \cos 2x$ (please, write justifications for all your steps).

3. (a) Write as the sum/difference of two functions
$$\sin x \sin 5x =$$

(b) Write as the product of two functions
$$\sin 3\theta + \sin 7\theta =$$

(c) Verify the identity $\sin 3x - \sin x = 2 \sin x - 4 \sin^3 x$ (please, write justifications for all your steps).

4. (a) Calculate precisely (by hand) $\cos^{-1}(\cos\frac{5\pi}{4})$.

(b) Do the same for $\sin\left(\sin^{-1}\frac{2}{3} + \cos^{-1}\frac{1}{5}\right)$

5. (a) Solve the equation $2 \sin x \cos x - \cos x = 0$, where $0 \leq x < 2\pi$.

(b) Solve the equation $2 \sin^2 x - 3 \sin x + 1 = 0$, where $0 \leq x < 2\pi$.