QUIZ 5 - MATH 131 Friday, February 26

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Read each problem **very carefully** before starting to solve it. Each problem is worth 5 points. It is necessary to show **all** your work. Correct answers without explanations are worth 0 points. GOOD LUCK!!

1. Suppose $\cos\alpha = -\frac{1}{3}$ and $\sin\beta = -\frac{3}{7}$, where $\frac{\pi}{2} < \alpha < \pi$ and $\frac{3\pi}{2} < \beta < 2\pi$. Calculate the numbers $\sin(\alpha+\beta)$ and $\cos(\alpha-\beta)$.

2. Suppose $\tan\alpha = -\frac{7}{2}$, with $90^\circ < \alpha < 180^\circ$. Calculate $\sin\frac{\alpha}{2}$.

3. Verify the identity

$$\cos 3x + \cos x = 4\cos^3 x - 2\cos x.$$

You must write on the margin which identity you are using at each step.

Table of Identities:

Sum Identities:

$$\begin{aligned}
\sin(\alpha+\beta) &= \sin\alpha\cos\beta + \sin\beta\cos\alpha \\
\cos(\alpha+\beta) &= \cos\alpha\cos\beta - \sin\alpha\sin\beta \\
\tan(\alpha+\beta) &= \frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}
\end{aligned}$$

Double-Angle Identities & Power-Reducing Identities:

From the Sum Identities you can figure these out easily. Recall that the technique involves taking β to be equal to α.

Half-Angle Identities:

$$\begin{aligned}
\sin\frac{\alpha}{2} &= \pm\sqrt{\frac{1-\cos\alpha}{2}} \\
\cos\frac{\alpha}{2} &= \pm\sqrt{\frac{1+\cos\alpha}{2}}
\end{aligned}$$

Product-to-Sum Identities:

$$\begin{aligned}
\sin\alpha\cos\beta &= \frac{1}{2}\left[\sin(\alpha+\beta)+\sin(\alpha-\beta)\right] \\
\cos\alpha\sin\beta &= \frac{1}{2}\left[\sin(\alpha+\beta)-\sin(\alpha-\beta)\right] \\
\cos\alpha\cos\beta &= \frac{1}{2}\left[\cos(\alpha+\beta)+\cos(\alpha-\beta)\right] \\
\sin\alpha\sin\beta &= \frac{1}{2}\left[\cos(\alpha-\beta)-\cos(\alpha+\beta)\right]
\end{aligned}$$

Sum-to-Product Identities:

$$\begin{aligned}
\sin x + \sin y &= 2\sin\frac{x+y}{2}\cos\frac{x-y}{2} \\
\cos x + \cos y &= 2\cos\frac{x+y}{2}\cos\frac{x-y}{2} \\
\sin x - \sin y &= 2\cos\frac{x+y}{2}\sin\frac{x-y}{2} \\
\cos x - \cos y &= -2\sin\frac{x+y}{2}\sin\frac{x-y}{2}
\end{aligned}$$