EXAM 4 - MATH 310 — Friday, April 16

YOUR NAME:________________________ — George Voutsadakis

Read each problem **very carefully** before starting to solve it. Each problem is worth 10 points. It is necessary to show **all** your work. Correct answers without explanations are worth 0 points. GOOD LUCK!!

1. Consider the 2×2 system $\left\{ \begin{array}{rcl} 7x_1 + 5x_2 & = & 20 \\ 4x_1 + 3x_2 & = & 11 \end{array} \right\}$.

   (a) Rewrite the system as a single matrix equation in the form $\boldsymbol{Ax} = \boldsymbol{b}$, where $\boldsymbol{x} = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$.

   (b) Find $\boldsymbol{A}^{-1}$.

   (c) Apply matrix multiplication on the left by $\boldsymbol{A}^{-1}$ to solve the system.

2. Solve the initial value problem $\boldsymbol{y}' = \begin{pmatrix} -5 & 1 \\ 4 & -2 \end{pmatrix} \boldsymbol{y}$, with $\boldsymbol{y}(0) = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$.

3. Solve the differential equation $\boldsymbol{y}' = \begin{pmatrix} -2 & 6 \\ -3 & 4 \end{pmatrix} \boldsymbol{y}$ and leave your answer in real form.

4. Consider a homogeneous system of first-order differential equations $\boldsymbol{y}' = A\boldsymbol{y}$. Suppose that we are given that, at $t_0 = 0$, the special fundamental matrix is

$$\boldsymbol{\Phi} = \begin{pmatrix} -2e^{-t} + 3e^{7t} & -2e^{-t} + 2e^{7t} \\ 5e^{-t} - 5e^{7t} & 2e^{-t} - e^{7t} \end{pmatrix}.$$

(a) Write a general solution of the given system $\boldsymbol{y}' = A\boldsymbol{y}$.

(b) Write a particular solution of the initial value problem

$$\boldsymbol{y}' = A\boldsymbol{y}, \quad \boldsymbol{y}(0) = \begin{pmatrix} 1 \\ -2 \end{pmatrix}.$$

5. Solve the differential equation $\boldsymbol{y}' = \begin{pmatrix} -2 & 1 \\ -1 & 0 \end{pmatrix} \boldsymbol{y}$ and leave your answer in real form.