

YOUR NAME: \_\_\_\_\_

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Read each problem **very carefully** before starting to solve it. Each problem is worth around 5 points. It is necessary to show **all** your work. Correct answers without explanations are worth 0 points. GOOD LUCK!!

1. (a) Use the cofactor method to find the inverse  $\mathbf{A}^{-1}$  of  $\mathbf{A} = \begin{pmatrix} 2 & -3 \\ 4 & -5 \end{pmatrix}$ . Please, show all steps; do not simply provide the result.

- (b) Use Gaussian elimination to find the inverse  $\mathbf{A}^{-1}$  of  $\mathbf{A} = \begin{pmatrix} 1 & 2 \\ -1 & 3 \end{pmatrix}$ . Please show all steps; do not simply give the result.

2. The product rule for derivatives of matrix functions states that, given matrices  $\mathbf{A}(t)$  and  $\mathbf{B}(t)$ ,  
$$\frac{d}{dt}(\mathbf{A}\mathbf{B}) = \frac{d\mathbf{A}}{dt}\mathbf{B} + \mathbf{A}\frac{d\mathbf{B}}{dt}.$$
 Suppose  $\mathbf{A}(t) = \begin{pmatrix} \sin t & t \\ 1 & \cos t \end{pmatrix}$  and  $\mathbf{B}(t) = \begin{pmatrix} e^{3t} & -t \\ t^2 & e^{-2t} \end{pmatrix}$ .

(a) Compute  $\mathbf{A}\mathbf{B}$ .

(b) Compute  $\frac{d}{dt}(\mathbf{A}\mathbf{B})$ .

(c) Compute  $\frac{d\mathbf{A}}{dt}$ .

(d) Compute  $\frac{d\mathbf{B}}{dt}$ .

(e) Compute  $\frac{d\mathbf{A}}{dt}\mathbf{B} + \mathbf{A}\frac{d\mathbf{B}}{dt}$ .