EXAM 2 - MATH 112 Friday, March 4

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Read each problem **very carefully** before starting to solve it. Each problem is worth 10 points. It is necessary to show **all** your work. Correct answers without explanations are worth 0 points. GOOD LUCK!!

1. Compute the following derivatives:

   (a) $\left(3x^5 - 4x^3 + 17x - 2022\right)' =$

   (b) $\left[(x^3 - 7x)^5 (2x+3)^{15}\right]' =$

2. Find an equation for the tangent line to the graph of $f(x) = \sqrt[3]{x^2 + 20x}$ at $x = 5$.

3. Consider the function $f(x) = x^4 - 4x^3$.

(a) Compute f' and find its critical points.

(b) Compute f'' and find its critical points.

(c) Create the combined signed table for f' and f'' as shown in class.

(d) Summarize the following information:

(i) f in increasing on:

(ii) f has a relative max/min points at:

(iii) f is concave down on:

(iv) f has inflection points at:

4. A moving object starting at the origin at time $t = 0$ covers distance $s(t) = \dfrac{t^2}{t+1}$ feet in t seconds.

   (a) Find the velocity of the object at time $t = 2$ seconds.

   (b) Find the acceleration of the object at time $t = 1$ second.

5. A triangle is formed so that the sum of three times its base plus its height has length 54 inches. Our goal is to find the length of the base that maximizes the triangle's area.

   (a) Make a small figure and set your variables (no variables that are not introduced and explained here may be used in subsequent parts).

   (b) Based on the variables you set in Part (a), give an equation for the objective function (i.e., the quantity to be maximized).

   (c) If the equation in Part (b) contains more than a single variable, use an auxiliary equation to eliminate one of the two variables.

   (d) Perform the optimization step on the objective function (which should now involve only a single variable) to find the length of the base that maximizes the triangle's area.