

YOUR NAME: _____

George Voutsadakis

Read each problem **very carefully** before starting to solve it. Each problem is worth 10 points. It is necessary to show **all** your work. Correct answers without explanations are worth 0 points. GOOD LUCK!!

1. Suppose an oil producing country can sell 30 million barrels of oil at a price of \$120 per barrel. Each \$10 price increase results in a sales decrease of 2 million barrels. Suppose, in addition that it costs the country \$10 to produce a barrel of oil. Let z denote the number of \$10 price increases per barrel.

- (a) Write an equation for the price per barrel as a function of z .

$$p(z) =$$

- (b) Write an equation for the number of barrels (in millions, to keep the numbers small) sold as a function of z .

$$q(z) =$$

- (c) Write equations for the revenue, the cost and the profit as functions of z .

$$R(z) =$$

$$C(z) =$$

$$P(z) =$$

- (d) Find which price per barrel would maximize the country's profit.

2. Find an equation for the tangent line to the graph of $x^3 - 5xy^2 + 3y^2 = -7x$ at the point $(x, y) = (3, -2)$.

3. A company finds that its profit from selling x units of a product is $P = x^3 - 20x^2 + 500x$ dollars. If the sales are increasing by 10 units per week, how fast is the profit increasing when 50 units have been sold?

4. (a) Solve the logarithmic equation $\log_3 x + \log_3 (x + 3) = \log_3 (2x + 6)$.

(b) How many years will it take for a deposit of \$10,000 in an account yielding 3% annually compounded monthly to increase to \$12,000?

5. (a) Compute the following derivatives:

$$\left(x^2 e^{x^3+7x}\right)' =$$

$$\left(\frac{\ln x}{x+5}\right)' =$$

(b) Find an equation for the tangent line to the graph of $f(x) = e^{-x^2} \ln(2x+1)$ at $x = 1$.
(If you use decimals, provide the exact expressions you plugged in the calculator to compute them!)

