

YOUR NAME: _____

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Read each problem **very carefully** before starting to solve it. Each problem is worth 10 points. It is necessary to show **all** your work. Correct answers without explanations are worth 0 points. GOOD LUCK!!

1. (a) Find an equation for the line that passes through the point $(-10, -1)$ and is perpendicular to the line passing through $(0, 10)$ and $(20, 5)$.

- (b) Consider the quadratic function $f(x) = -x^2 + 6x - 8$. Answer the following questions “by hand”, showing all work.

Find the location of the vertex.

State the opening direction, with a justification.

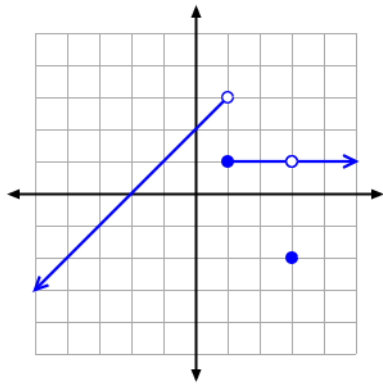
Find the y -intercept.

Find the x -intercepts.

2. (a) Find the domain of the function $f(x) = \frac{x^2 - 4}{x^3 - 4x^2 - 5x}$.

(b) Given $f(x) = \sqrt{x^2 + 2x}$ and $g(x) = x - 2$, find a formula for the composite $(f \circ g)(x)$ and simplify.

3. Let $f(x)$ be the function whose graph is shown below. Identify the following:



$$f(1) =$$

$$f(3) =$$

$$\lim_{x \rightarrow 1^-} f(x) =$$

$$\lim_{x \rightarrow 3^-} f(x) =$$

$$\lim_{x \rightarrow 1^+} f(x) =$$

$$\lim_{x \rightarrow 3^+} f(x) =$$

$$\lim_{x \rightarrow 1} f(x) =$$

$$\lim_{x \rightarrow 3} f(x) =$$

4. Calculate the following limit algebraically.

$$\lim_{x \rightarrow 5} \frac{\frac{1}{x-2} - \frac{1}{3}}{5-x} =$$

5. Consider the piecewise defined function

$$f(x) = \begin{cases} \frac{x^2 - 6x + 5}{x^2 - 4x + 3}, & \text{if } x < 1 \\ -\sqrt{x} + 5, & \text{if } x \geq 1 \end{cases} .$$

Find the following

$$f(1) =$$

$$\lim_{x \rightarrow 1^-} f(x) =$$

$$\lim_{x \rightarrow 1^+} f(x) =$$

From the following three statements, circle all that apply:

f is left continuous at $x = 1$ f is right continuous at $x = 1$ f is continuous at $x = 1$