

YOUR NAME: _____

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Read each problem **very carefully** before starting to solve it. Each problem is worth 10 points. It is necessary to show **all** your work. Correct answers without explanations are worth 0 points. GOOD LUCK!!

1. Use the limit definition of the derivative to find $f'(1)$ if $f(x) = \frac{1}{5x - 2}$.

2. Compute the following derivatives

$$\left(x^{11} - 3\sqrt[3]{x} + \frac{7}{\sqrt[7]{x^2}}\right)' =$$

$$[(x^2 + 4)^3(5x - 1)^7]' =$$

3. Find an equation for the tangent line to $f(x) = \frac{7x + 4}{2x - 1}$ at $x = 3$.

4. A population of bacteria in a contaminated culture is modeled by

$$P(t) = 2 + \frac{50}{t^2 + 1}$$

in hundreds of individuals, where t is the day after the experiment began. Find the rate of change of the population three days after the start of the experiment (please, provide units).

5. Consider the polynomial function

$$f(x) = -x^3 + 3x^2 + 9x.$$

(a) Find $f'(x)$ and the critical points.

(b) Create the sign table of the first derivative and summarize your findings regarding intervals of monotonicity (on which function is increasing/decreasing) and relative extrema (maxima and minima) in the last line of the table.

(c) Use the information shown on the table of Part (b) to roughly sketch the graph of $y = f(x)$. Be as neat as you can and make sure to label all important points.