EXAM 3 - MATH 112

YOUR NAME:⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯

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Read each problem **very carefully** before starting to solve it. Each problem is worth 10 points. It is necessary to show **all** your work. Correct answers without explanations are worth 0 points. GOOD LUCK!!

1. Consider the function $f(x) = x^3 - 9x^2 + 108$.

    (a) Compute $f'(x)$ and find the critical points.

    (b) Compute $f''(x)$ and find critical points.

    (c) Create the combined sign table for f' and f''.

    (d) Sketch the graph of $y = f(x)$ putting together all information gathered in the last line of the sign table.

2. A grandma bought 300 ft of fencing and wants to enclose two identical rectangular gardens as shown in the figure, where the side along the wall does not need to be fenced. Find the dimensions of each individual garden that maximizes the total area enclosed.

WALL

y

x
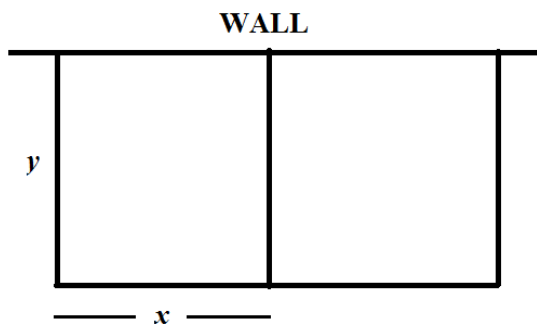
Objective Function:

Auxiliary Equation:

Optimization Step:

3. Suppose a toy maker sells 100 toys per month at the price of \$40 each. She has discovered that, for every \$2 reduction in price, she is going to sell 10 more toys per month. Suppose x is the number of \$2 reductions she decides to make.

   (a) Write an equation for the price $p(x)$ as a function of x.

   (b) Write an equation for the quantity $q(x)$ of toys sold per month as a function of x.

   (c) Write an equation of the monthly revenue $R(x)$ of her manufacturing operation as a function of x.

   (d) Find the price at which each toy should be sold to maximize the revenue.

4. Find an equation for the tangent line to

$$y^2 + x^2y^3 = 10$$

at the point $(3, 1)$.

5. Suppose that the profit of an investments firm as a function of the number x of its clients is given by $P(x) = 2x^3 + 3x$ in thousands of dollars. If the firm is losing 10 customers per month, find the rate at which its profit is diminishing when the firm has exactly 100 clients.

(Hint: This is a related rates problem. So work with the triangle of dependencies.)