Read each problem **very carefully** before starting to solve it and do only what is asked. Each problem is worth around 5 points. It is necessary to show **all** your work. Correct answers without explanations are worth 0 points. GOOD LUCK!!

1. [5 points] Solve the initial value problem

$$xy' = (6x^3 + 2x^2)y, \quad y(1) = 5e^3, \ y > 0.$$

2. [4 points] Find the following limits, justifying all your steps (do not simply give the answers).

(a) $\lim\limits_{n\to\infty} \left[\dfrac{n^2}{n^2 + \ln n} \right] =$

(b) $\lim\limits_{n\to\infty} \left[5 \left(-\dfrac{2}{3} \right)^n \right] =$

3. [5 points] Prove formally that the sequence $a_n = \sqrt{n-1} - \sqrt{n+1}$, for $n \geq 1$, converges. Give all details. (Hint: We did one very similar in class!)