EXAM 1 - MATH 310 Thursday, February 9

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Read each problem **very carefully** before starting to solve it. Each problem is worth 10 points. It is necessary to show **all** your work. Correct answers without explanations are worth 0 points. GOOD LUCK!!

1. Solve the initial value problem

$$\frac{dy}{dx} = \frac{x^2}{(x^3+1)(y^2-3)}, \quad y(0) = 1 \quad (x \geq 0).$$

2. Find the general solution of the differential equation

$$\frac{1}{x}y' - \frac{2}{x^2}y = x\cos x, \quad x > 0.$$

3. A 500-liter tank initially contains 10 g of salt dissolved in 200 liters of water. Starting at $t_0 = 0$, water that contains 1/4 g of salt per liter is poured into the tank at the rate of 4 liters/min and the mixture is drained from the tank at the rate of 2 liters/min. Find the amount of salt $Q(t)$ in the tank at time t prior to the time when the tank overflows.

4. Check whether the given equation is exact and, if yes, solve it.

$$\left(\frac{y}{x} + 6x\right) + (\ln x - 2)\,\frac{dy}{dx} = 0, \quad y(1) = 5, \quad x > 0.$$

5. Solve the initial value problem

$$y'' - 2y' - 3y = 0, \quad y(0) = 4, \quad y'(0) = 6.$$