Read each problem very carefully before starting to solve it. Each problem is worth 10 points. It is necessary to show all your work. Correct answers without explanations are worth 0 points. GOOD LUCK!!

1. Solve the initial value problem

$$
4 y^{\prime \prime}+12 y^{\prime}+9 y=0, \quad y(0)=14, y^{\prime}(0)=-2 .
$$

2. Solve the initial value problem

$$
4 y^{\prime \prime}+24 y^{\prime}+37 y=0, \quad y(0)=1, \quad y^{\prime}(0)=-2 .
$$

3. Find the general solution of

$$
y^{\prime \prime}+3 y^{\prime}+2 y=-14 e^{-2 t} .
$$

4. Find the general solution of

$$
y^{\prime \prime}+2 y^{\prime}+y=25 \cos 3 t .
$$

5. Consider the second order, linear, homogeneous differential equation, with non-constant coefficients,

$$
(t-1) y^{\prime \prime}-t y^{\prime}+y=0, \quad t>1 .
$$

(a) Verify that $y_{1}(t)=e^{t}$ is a solution.
(b) Find a differential equation that $v(t)$ should satisfy so that $y_{2}(t)=v(t) y_{1}(t)$ is also a solution.
(c) Solve the differential equation of Part (b) to find $y_{2}(t)$.

