

YOUR NAME: \_\_\_\_\_

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Read each problem **very carefully** before starting to solve it. Each problem is worth 10 points. It is necessary to show **all** your work. Correct answers without explanations are worth 0 points. GOOD LUCK!!

1. Solve the differential equation  $y^{(4)} - y = -36e^{-t}$ .

2. In Parts (a) and (b), show how the following formulas are obtained from scratch:

(a)  $\mathcal{L}\{f'(t)\} = s\mathcal{L}\{f(t)\} - f(0)$

(b)  $\mathcal{L}\{e^{ct}f(t)\} = F(s - c)$ , where  $F(s) = \mathcal{L}\{f(t)\}$ .

3. Use Laplace transforms to solve the initial value problem

$$y^{(4)} + 4y'' = 0, \quad y(0) = 2, \quad y'(0) = -2, \quad y''(0) = -12, \quad y'''(0) = 16.$$

4. Consider the piece-wise defined function  $f(t) = \begin{cases} -1, & \text{if } 0 \leq t < 2 \\ 6, & \text{if } 2 \leq t < 5 \\ 1, & \text{if } t \geq 5 \end{cases}$ .

(a) Sketch the graph  $y = f(t)$ .

(b) Express  $f(t)$  using unit step functions.

(c) Use the Laplace transform table to find  $\mathcal{L}\{f(t)\}$ .

5. Solve the initial value problem

$$y'' + 2y = u_3(t), \quad y(0) = 0, \quad y'(0) = 0.$$