

YOUR NAME: _____

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Read each problem **very carefully** before starting to solve it. Each problem is worth 10 points. It is necessary to show **all** your work. Correct answers without explanations are worth 0 points. GOOD LUCK!!

1. (a) Express the function $g(t) = \begin{cases} \sin t, & \text{if } 0 \leq t < \pi \\ 0, & \text{if } t \geq \pi \end{cases}$ in terms of unit step functions.

- (b) Find the solution of the given initial value problem, where $g(t)$ is the function given in Part (a).

$$y'' + 3y = g(t), \quad y(0) = 0, \quad y'(0) = 0.$$

2. Find the solution of the initial value problem

$$y'' + y = \delta(t - 2\pi) \cos t, \quad y(0) = 0, \quad y'(0) = 1.$$

(**Hint:** Recall that $\int_{-\infty}^{\infty} \delta(t - t_0) f(t) dt = f(t_0)$.)

3. Express the solution of the given initial value problem in terms of a convolution integral

$$y'' + 4y' + 4y = g(t), \quad y(0) = 2, \quad y'(0) = -3.$$

4. (a) Find the eigenvalues and corresponding eigenvectors of the matrix $A = \begin{pmatrix} 2 & -1 \\ 3 & -2 \end{pmatrix}$.

- (b) Find the general solution of the system of differential equations

$$\begin{cases} x_1'(t) = 2x_1(t) - x_2(t) \\ x_2'(t) = 3x_1(t) - 2x_2(t) \end{cases}.$$

5. Solve the given initial value problem:

$$\mathbf{x}' = \begin{pmatrix} -2 & 1 \\ -5 & 4 \end{pmatrix} \mathbf{x}, \quad \mathbf{x}(0) = \begin{pmatrix} 1 \\ 3 \end{pmatrix}.$$