Friday, March 29 George Voutsadakis

Read each problem **very carefully** before starting to solve it. Each problem is worth 10 points. It is necessary to show **all** your work. Correct answers without explanations are worth 0 points. GOOD LUCK!!

1. (a) Perform the long division $(x^4 - x^3 - 5x^2 - 3) \div (x^2 + 4)$ and write your answer in the appropriate form.

(b) Let $f(x) = 3x^3 - 5x^2 - 107x - 35$. Given that f(7) = 0, use the factor theorem to factor f(x) completely and find all its remaining roots.

- 2. Consider the rational function $f(x) = \frac{(x+2)(x-5)}{(x+5)(x-1)^2}$.
 - (a) Find the domain.
 - (b) Find the vertical asymptotes.
 - (c) Find the horizontal asymptote (all steps).
 - (d) Find the *y* and the *x*-intercepts (all steps).
 - (e) Sketch the graph by hand, showing all important features.



3. Consider the graph of a rational function y = f(x) shown below.

- (a) Find the domain.
- (b) Find the vertical asymptotes.
- (c) Find the horizontal asymptote.
- (d) Find the y- and the x-intercepts.
- (e) Find a formula for y = f(x) with all explanations.

4. A quantity x varies directly with the cube root of y and inversely with the square of z. Suppose we know that, when y = 64 and z = 2, then x = 5. Find the value of y, when x = 5 and z = 2 (Show all steps).

5. In a park, the initial population of a certain species consisted of 136 individuals. Three years later the population had fallen to only 17 individuals. Assuming an exponential trend, find (by hand, showing all steps) a model for the population P(t) as a function of time t.