## EXAM 3 - MATH 111 YOUR NAME:

Read each problem very carefully before starting to solve it. Each problem is worth 10 points. It is necessary to show all your work. Correct answers without explanations are worth 0 points. GOOD LUCK!!

1. (a) Perform the long division $\left(x^{4}-x^{3}-5 x^{2}-3\right) \div\left(x^{2}+4\right)$ and write your answer in the appropriate form.
(b) Let $f(x)=3 x^{3}-5 x^{2}-107 x-35$. Given that $f(7)=0$, use the factor theorem to factor $f(x)$ completely and find all its remaining roots.
2. Consider the rational function $f(x)=\frac{(x+2)(x-5)}{(x+5)(x-1)^{2}}$.
(a) Find the domain.
(b) Find the vertical asymptotes.
(c) Find the horizontal asymptote (all steps).
(d) Find the $y$ - and the $x$-intercepts (all steps).
(e) Sketch the graph by hand, showing all important features.
3. Consider the graph of a rational function $y=f(x)$ shown below.

(a) Find the domain.
(b) Find the vertical asymptotes.
(c) Find the horizontal asymptote.
(d) Find the $y$ - and the $x$-intercepts.
(e) Find a formula for $y=f(x)$ with all explanations.
4. A quantity $x$ varies directly with the cube root of $y$ and inversely with the square of $z$. Suppose we know that, when $y=64$ and $z=2$, then $x=5$. Find the value of $y$, when $x=5$ and $z=2$ (Show all steps).
5. In a park, the initial population of a certain species consisted of 136 individuals. Three years later the population had fallen to only 17 individuals. Assuming an exponential trend, find (by hand, showing all steps) a model for the population $P(t)$ as a function of time $t$.
