YOUR NAME:_____                      George Voutsadakis

Read each problem **very carefully** before starting to solve it. Each problem is worth 10 points. It is necessary to show **all** your work. Correct answers without explanations are worth 0 points. GOOD LUCK!!
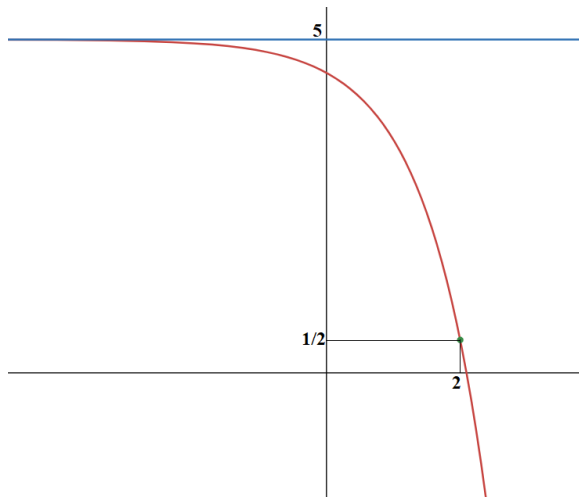
1.  (a) Detect the transformations that lead from the graph of $f(x) = 2^x$ to the graph of $g(x) = 2^{3x+1} + 5$.

$$y = 2^x \quad \longrightarrow \qquad\qquad\qquad ( \qquad\qquad\qquad )$$

$$\longrightarrow \qquad\qquad\qquad ( \qquad\qquad\qquad )$$

$$\longrightarrow \quad y = 2^{3x+1} + 5 \qquad ( \qquad\qquad\qquad )$$

(b) The following is a the graph of a transform $y = f(x)$ of the exponential function with base 3. Find a possible formula for $y = f(x)$.

2. Consider the function $f(x) = \log_{\frac{1}{2}} x$.

   (a) Use a small table of values to sketch the graph of $y = f(x)$.

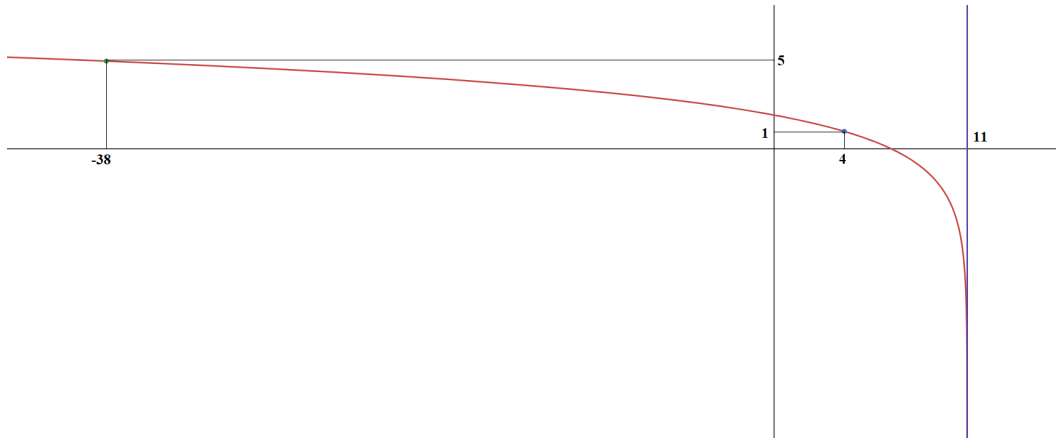   (b) Detect the transformations that lead from $f(x)$ to $g(x) = 3\log_{\frac{1}{2}}(x+1) - 2$.

   $$y = \log_{\frac{1}{2}} x \quad \longrightarrow \qquad\qquad ( \qquad\qquad\qquad )$$

   $$\longrightarrow \qquad\qquad ( \qquad\qquad\qquad )$$

   $$\longrightarrow \quad y = 3\log_{\frac{1}{2}}(x+1) - 2 \quad ( \qquad\qquad\qquad )$$

   (c) Use Parts (a) and (b) to sketch the graph of $y = g(x)$ clearly labeling all important features.

3. The following graph shows a transform of the logarithm to base 7. Find a possible formula for it.

5

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4. Fully expand or contract, as appropriate, showing all steps.

$$\log\left(\frac{7(x+2)^3}{\sqrt{y}(z+2)}\right) =$$

$$7 \ln x - \tfrac{1}{3} \ln y + 2 \ln (z + 1) =$$

5. Solve the following equations:

   (a) $10e^{8x+3} + 2 = 37$

   (b) $\log x + \log (x - 3) = 1.$