EXAM 3 - MATH 310   Thursday, March 28
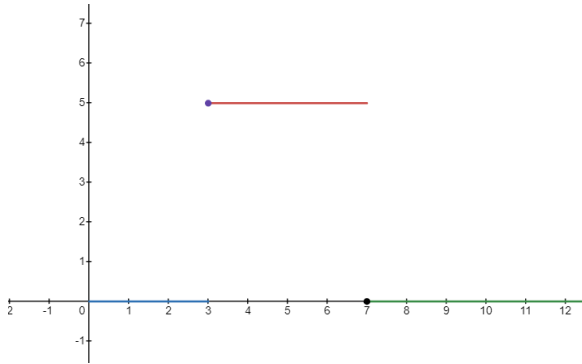## YOUR NAME:_____   George Voutsadakis

Read each problem **very carefully** before starting to solve it. Each problem is worth 10 points. It is necessary to show **all** your work. Correct answers without explanations are worth 0 points. GOOD LUCK!!

1. Consider the function whose graph is shown below.

(a) Express $y = f(x)$ as a piecewise defined function and compute its Laplace transform from scratch.

(b) Express $y = f(x)$ in terms of unit step functions and compute its Laplace transform using the table.

2. Suppose we know $\mathcal{L}\{f\} = F(s)$.

   (a) Prove that $\mathcal{L}\{f'\} = sF(s) - f(0)$.

   (b) Use Part (a) to find a formula for $\mathcal{L}\{f'''\}$ showing all steps.

3. Find the inverse Laplace transform of $F(s) = \dfrac{8s + 3}{s(s + 1)(s^2 + 4)}$.

4. Use Laplace transforms to solve the initial value problem

$$y'' + 4y = 3 + 5e^{-t}, \quad y(0) = 0, \quad y'(0) = 0.$$

5. Use Laplace transforms to solve the initial value problem

$$y'' + 4y' + 3y = f(t), \quad y(0) = 0, \quad y'(0) = 1, \quad f(t) = \begin{cases} 1, & \text{if } 0 \le t < 5, \\ 0, & \text{if } t \ge 5. \end{cases}$$