EXAM 4 - MATH 310 Thursday, April 18

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Read each problem **very carefully** before starting to solve it. Each problem is worth 10 points. It is necessary to show **all** your work. Correct answers without explanations are worth 0 points. GOOD LUCK!!

1. Consider the function $f(x) = \begin{cases} -2, & \text{if } 0 \leq t < 3, \\ 2e^{3-t}, & \text{if } t \geq 3 \end{cases}$

   (a) Compute $\mathcal{L}\{f\}$ from scratch.

   (b) Express $y = f(x)$ in terms of unit step functions and compute its Laplace transform using the table.

2. Solve the initial value problem

$$y'' - y = f(t), \quad y(0) = 0, \quad y'(0) = 2,$$

where $f(t)$ is the function given in Problem 1.

(No hyperbolic functions are allowed in the solution.)

3. Consider the function $f(t) = d_\tau(t-5)$.

   (a) Graph $y = f(t)$ and write a piece-wise expression for $f(t)$.

   (b) Express $f(t)$ in terms of unit step functions and find $\mathcal{L}\{f\}$ using the table.

   (c) Use Part (b) (without further recourse to tables) to find $\mathcal{L}\{\delta(t-5)\}$.

4. Use Laplace transforms to solve the initial value problem

$$y'' + 14y' + 49y = 3 + 5\delta(t-2), \quad y(0) = 0, \quad y'(0) = 0.$$

5. Use Laplace transforms to solve the initial value problem

$$y'' - 6y' + 25y = g(t), \quad y(0) = 0, \quad y'(0) = 1.$$

(Of course, the answer must be expressed using a convolution integral.)