Read each problem **very carefully** before starting to solve it and do only what is asked. Each problem is worth around 5 points. It is necessary to show **all** your work. Correct answers without explanations are worth 0 points. GOOD LUCK!!

1. [6 points] A container contains initially a solution consisting of 50 grams of salt in 10 liters of water. Incoming at the rate of 1 liter/hour is a solution containing 30 grams of salt per liter. The container is drained also at the rate of 1 liter/hour. Write an initial value problem for the quantity y(t) of salt in the container at time t.

Differential Equation:

Initial Condition:

Solve the initial value problem to find how much salt will be in the container at time t.

2. [4 points] Consider the initial value problem

$$\left(\frac{1}{x} + 2y^2x\right) + (2x^2y - \cos y)\frac{dy}{dx} = 0, \quad y(1) = \pi.$$

(a) Verify that the given differential equation is exact.

(b) Solve the initial value problem.

3. [4 points] Consider the differential equation

$$(3x^2y + 2xy + y^3) + (x^2 + y^2)\frac{dy}{dx} = 0.$$

(a) Is the given differential equation exact?

(b) If it is, solve it. If it is not, find a function  $\mu(x)$  that, upon multiplication, makes it exact. In the latter case, you do not have to solve.