Thursday, March 27 George Voutsadakis

Read each problem **very carefully** before starting to solve it. Each problem is worth 10 points. It is necessary to show **all** your work. Correct answers without explanations are worth 0 points. GOOD LUCK!!

1. (a) Compute the indefinite integral  $\int t e^{-st} dt$ .

(b) Consider the function

$$f(t) = \begin{cases} t, & \text{if } 0 \le t < 1, \\ 2 - t, & \text{if } 1 \le t < 2, \\ 0, & \text{if } t \ge 2. \end{cases}$$

Compute from scratch the Laplace transform  $\mathcal{L}{f}$ . (You may use Part (a).)

2. Compute the inverse Laplace transform of

$$F(s) = \frac{2s^2 + 8s + 10}{(s+1)^2(s+2)}.$$

3. Compute the Laplace transform F(s) of the particular solution of the initial value problem

$$y'' + 3y' + 2y = 4e^{-t}, \quad y(0) = 2, \quad y'(0) = 0.$$

Express F(s) as a single fraction with completely factored denominator.

(You do not have to compute y = f(t), unless, of course, you already have it!)

4. (a) Find the Laplace transform of

$$f(t) = (t-3)u_2(t) - (t-2)u_3(t).$$

(b) Find the inverse Laplace transform of

$$F(s) = \frac{(s+7)e^{-2s}}{s^2 - 2s + 5}.$$

5. Solve the initial value problem

$$y'' + y = u_{3\pi}(t), \quad y(0) = 1, \quad y'(0) = 0.$$